Continuous control of ionization wave chaos by spatially derived feedback signals

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Abstract

In the positive column of a neon glow discharge, two different types of ionization waves occur simultaneously. The low–dimensional chaos arising from the nonlinear interaction between the two waves is controlled by a continuous feedback technique. The control strategy is derived from the time–delayed autosynchronization method. Two spatially displaced points of observation are used to obtain the control information, using the propagation characteristics of the chaotic wave.

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Due to their inherent nonlinearity, instabilities in plasmas often develop towards chaotic dynamics and turbulence [1]. In many practical cases this is considered as an undesired situation and there is a particular interest to influence the plasma system in order to achieve a stationary state (fixed point in the phase space) or a state of regular motion (limit cycle in the phase space). The most straightforward approach would be to change the set of discharge parameters to establish a new non–chaotic state. This, of course, may be cumbersome or even impossible. Recent results [2] have demonstrated the efficiency of chaos control in laboratory plasmas. Moreover, recent computational studies of chaos control strategies [3] offer the possibility of applications in fusion plasmas. In the present communication we present a simple continuous feedback technique to control chaotic states in plasmas arising from the nonlinear interaction of waves. The efficiency of the control scheme is demonstrated experimentally for ionization wave chaos in the positive column of a glow discharge.

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Typically, an infinite number of unstable periodic orbits (UPOs) is embedded within a chaotic phase space attractor [4]. This observation motivates the idea to achieve the stabilization of selected UPOs by means of small time dependent perturbations of an accessible control parameter. Ott, Grebogi, and Yorke (further referred to as OGY) have proposed an elegant control strategy based on the stabilization of fixed points in the Poincaré section [5] that has achieved a broad field of practical applications [6]. The main obstacle for a universal use of OGY to control chaotic states is the necessity for the online determination of the Poincaré mapping. Therefore, in non–driven (autonomous) chaotic systems the OGY scheme is limited by computational speed. In such situations a control strategy based on information obtained in the time domain becomes advantageous. A particular method that uses the information of previously recorded dynamics to determine the required control information was proposed by Pyragas [7,8]. It is referred to as time–delay autosynchronization (TDAS) method. If, for simplicity, a two-dimensional dynamical system is considered, TDAS is described by the autonomous system [7]

$$\begin{aligned} \dot{Y} &= P(X,Y) + F_\tau(Y) \\ \dot{X} &= Q(X,Y)\,, \end{aligned} \tag{1}$$

where $\{X(t), Y(t)\}$ represents the state of the system. The unperturbed phase space flow is given by the functions P and Q. The control signal $F_\tau(Y)$ is obtained by the linear control law $F_\tau(Y) = K \cdot [Y(t-\tau) - Y(t)]$ where τ denotes an appropriate time delay. It is chosen equal to the fundamental period of the oscillation signal that corresponds to a particular UPO. The constant factor K determines the feedback strength and F_τ vanishes if control of the desired periodic orbit is achieved. The TDAS control method has been applied succesfully to experimental chaotic systems, for instance lasers [9], electronic circuits [8,10] and plasma discharges [2]. It was extended later for better performance [11,12] and its mechanism can be understood to some extent now [13]. TDAS and its variants meet technical limitations, too. The time–delayed signal $Y(t-\tau)$ is obtained either by delay lines [8–10] (fast systems) or digitally stored data [2] (slow systems). The subsequently discussed control scheme may overcome such technical problems.

Starting with a chaotic state, the wave character of dynamics allows us to stabilize unstable periodic orbits. Considering the fact that temporally periodic states are related to a wavenumber by the dispersion relation, the time delay τ may be replaced by a spatial displacement such that the condition $\omega/k = v_\varphi = \zeta/\tau$ holds,where τ is fixed by the period of the orbit to be stabilized. Here the delay is chosen as $\tau = \Delta z/v_\varphi$ with $\Delta z = n\lambda$. The resulting control law now reads $F_\zeta(Y) = K \cdot [Y(z-\zeta, t) - Y(z,t)]$. It is easily realized by using a differential amplifier and two spatially displaced detectors. K is a constant gain factor that may be determined experimentally. The control

signal $F_\zeta(Y)$ is applied to an accesible dynamical quantity.

For an experimental demonstration we have chosen the chaotic dynamics of ionization waves propagating in the positive column of an ordinary glow discharge tube. This system is a representative example for spatially extended dynamics that contain a wavetrain with a large number of wavelengths. In certain parameter regions of the pressure p and the discharge current I_d, the positive column is either homogenous or different types of ionization waves occur [14,15]. The nonlinear dynamics of ionization waves in glow discharges has already been investigated in great detail. Low–dimensional chaos was first observed in autonomous systems with the discharge current as control parameter [16,17]. In the non–autonomous case, where the discharge current is modulated by an external periodic driver signal, low–dimensional chaotic phase space attractors were studied in detail [18–21]. In the latter case, a simplified variant of the OGY scheme has already been successfully implemented and UPOs up to periodicity 32 could be stabilized [22]. Note, that the success of this approach relies on constructing the Poincaré section by making use of the periodic external driver signal. In the case of an autonomous chaotic system it was recently demonstrated in a discharge similar to the one under investigation that control could be achieved by a slightly modified variant of the TDAS scheme [2].

The experimental investigations are performed in a conventional cold-cathode glow discharge tube. The discharge length is $l = 600\,\mathrm{mm}$ and the tube has a radius of $r = 15\,\mathrm{mm}$. The ionization waves are observed by picking up the light emission flux $\Phi(z,t)$ with two movable optical fibres connected to fast photodiodes. The spatial resolution is estimated to be $\Delta z \leq 1\,\mathrm{cm}$, which is well below the typical wavelength of ionization waves. The discharge is operated at a pressure of $p = 1.8\,\mathrm{mbar}$ with neon as filling gas. The discharge current can be varied between $I_d = 1 \ldots 50\,\mathrm{mA}$. In this pressure range a positive column forms that extends over 80% of the discharge length. For the present discharge conditions two different types of ionization waves are observed simultaneously [23,24]: (I) p–waves (due to atomic ions) with a frequency of $f_p \approx 3\,\mathrm{kHz}$ and a phase velocity $v_{\varphi,p} = 247\,\mathrm{m/s}$ directed towards the cathode, and (II) s'–waves (due to metastables) with a frequency of $f_{s'} \approx 5.6\,\mathrm{kHz}$ and a phase velocity $v_{\varphi,s'} = 509\,\mathrm{m/s}$ directed towards the anode. Fig. 1 shows time series of the waves in a mode–locked state. Near the anode the p–wave dominates, whereas at the cathode a pure s'–wave appears. The frequency matching condition $f_{s'} = 2f_p$ is met in the midsection of the discharge tube. Since both wave types propagate simultaneously in the positive column, they show pronounced nonlinear interaction. Consequently a broad variety of dynamical phenomena and low–dimensional chaos is observed as mentioned above.

For the practical realization of the control scheme only few conditions have to be satisfied. Due to dispersion and the spatial amplification property of the

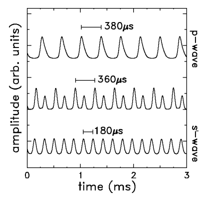

Fig. 1. Time series of the integral light emission fluctuations corresponding to the p–wave (top) and the s–wave (bottom). The time series are recorded at $z_p = 50\,\mathrm{mm}$ (close to anode) and $z_s = 580\,\mathrm{mm}$ (close to cathode). The waves are mode–locked where the frequency matching condition $f_s = 2f_p$ is met in the midsection of the discharge tube (center).

positive column, the amplitudes of the measured light fluctuations have to be equalized, so that the control signal depends only on the phase information. The spacing between the two optical fibres has to be exactly one wavelength or integer multiples thereof for an optimal control. This finding supports the interpretation that the two–point observation acts as a spatial filter. The axial position of both fibres, i.e. the distance from the anode is used to adjust the phase of the control signal.

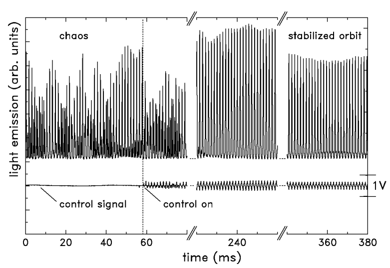

Fig. 2. Time series of the integral light emission fluctuations ($Y(z,t)$) and the control signal. The control is applied at $t_0 = 58\,\mathrm{ms}$ and the stabilization is achieved within $150\,\mathrm{ms}$. The modulation degree of the discharge current I_{mod}/I_{dc} does not exceed 5%. Smaller K-values as well as larger ones lead to a loss of control.

The stabilization of an UPO of a chaotic state arising from the interaction of two different ionization waves is demonstrated in Fig. 2. The control signal is applied at $t_0 = 58\,\mathrm{ms}$. After approximately $150\,\mathrm{ms}$ a periodic orbit of pe-

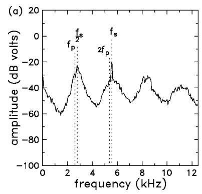

Fig. 3. In (a) the power spectrum of the time series is shown in the chaotic state. Both frequencies (f_p and f_s) according to the specific ionization waves are observable. In (b) the reconstructed phase space of the corresponding time series is shown. The chaotic attractor ($D_2 = 3.5$) is embedded into a smaller space of three dimensions. The cube axes correspond to a time lag of $\tau = 4$ between successive data points. The number of data points for the reconstruction of the phase space vectors is $N = 10000$.

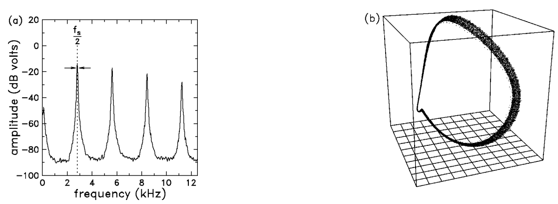

Fig. 4. In (a) the power spectrum of the time series is shown in the controlled state. Only f_s is remaining due to the stabilization of both waves. In (b) the reconstructed phase space of the corresponding time series is shown. Due to the reduction of attractor dimensionality ($D_2 = 1$) the embedding into the three–dimensional space is sufficient. The cube axes correspond to a time lag of $\tau = 4$ between successive data points. The number of data points for the reconstruction of the phase space vectors is $N = 10000$.

riodicity $P = 1$ is fully stabilized as long as the control signal is present. It was not possible to stabilize UPOs of higher periodicity. This is, however, an often recognized limitation of continuous feedback techniques [8]. The remaining modulation of the control signal in Fig. 2 is caused by small differences in the shape of the fluctuation signals due to the different observation points. The control method is nevertheless quite efficient since the open loop control would require a much higher amplitude for the suppression of chaos [25]. The application of control leads to a dramatic decrease (≈70dB) of broad–band components in the power spectrum [see Figs. 3a and 4a]. In the chaotic state, the two ionization waves occur as small maxima in a broad spectrum, whereas

in the controlled state only one frequency is established. The inspection of the reconstructed phase space diagrams [see Figs. 3b and 4b] illustrates the reduction of the attractor's dimensionality. The correlation dimension [26] of the chaotic state has been estimated to be $D_2 = 3.5 \pm 0.3$. The estimation of

Table 1
Lyapunov spectra and corresponding dimensionalities [27] for the chaotic and the controlled state.

state	λ_1	λ_2	λ_3	λ_4	D_{KY}
chaotic	0.34 ± 0.05	0.15 ± 0.05	0.06 ± 0.08	-0.74 ± 0.10	3.74 ± 0.33
stable	0.02 ± 0.03	-0.95 ± 0.10	–	–	1.02 ± 0.03

the spectrum of Lyapunov exponents and the corresponding Kaplan-Yorke dimension [27] (Table 1) was done with the computation program of Kruel and Eiswirth [28] based on the algorithm of Sano and Sawada [29]. It shows that two positive Lyapunov exponents are dominating the dynamics ($D_2 \simeq D_{KY}$). The uncertainty is estimated by comparing the results of several calculations. After the stabilization, the dimensionality is reduced to $D_2 = 1.0 \pm 0.2$ (limit cycle).

To summarize, the control of ionization wave chaos in a neon glow discharge by a continuous control technique where the feedback signal is derived from spatial displacement of detectors has been demonstrated experimentally. The control information is easy to determine, the feedback scheme is quite simple and even fast dynamical systems can be controlled in which digital electronic fails. In contrast to the TDAS–approach, no time–delay lines and phase shifting circuits are required. The optimum control signal depends sensitively on the precise distance between the optical fibres and the absolute z-position due to transit time effects and differing shapes of the wave in the positive column.

This control strategy is expected to be more efficient than earlier attempts to suppress chaos by a simple open loop control [25] and could be of major interest in the various chaotic situations frequently observed in plasmas.

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